# Polarization structure of quantum light fields: a new insight. I. General outlook



1993 J. Phys. A: Math. Gen. 26 4345

(http://iopscience.iop.org/0305-4470/26/17/040)

J. Phys. A: Math. Gen. 26 (1993) 4345–4354. Printed in the UK

# Polarization structure of quantum light fields: a new insight. 1: general outlook

V P Karassiov

Lebedev Physical Institute, Leninsky prospect 53, Moscow 117924, Russia

Received 9 September 1992

**Abstract.** A new consequent description of the polarization structure of light within the framework of quantum optics is given by using the polarization gauge SU(2) invariance of free electromagnetic fields and a related concept of the polarization ($P$) spin. Within this approach the quantum light may be thought as a mixture of the usual polarized photons and unpolarized biphotons. New classes of unpolarized light states generated by P-scalar biphotons are examined. Possible applications are briefly discussed.

## 1. Introduction

For the last several decades polarization properties of light were widely investigated in both theoretical and applied aspects (see [1–13] and references therein). Specifically, some fundamental problems of quantum mechanics, related to 'hidden' variables, Bell's inequalities and Einstein–Podolsky–Rosen (EPR) paradox, quantum chaos, Berry and other topological phases, etc, are intensively examined with the help of quantum polarization optics (see [1, 4, 5] and references therein). However, as a rule, the polarization structure of light has been described in terms of the field correlation functions and associated Stokes parameters which are well adapted to classical optics experiments [3] but are not quite adequate to specific quantum ones (photon counting) [2]. Such a description also ignores a polarization SU(2) symmetry [14–17] of light fields though it has been widely used implicitly—through the Stokes parameters $s_a$ which determine, in particular, the polarization degree

$$\deg P = [s_1^2 + s_2^2 + s_3^2]^{1/2}/s_0 \tag{1.1}$$

of monochromatic plane wave light beams [2, 3, 6, 18]. Furthermore, the physical meaning of the Stokes parameters and their connections with the spin properties of light fields are sufficiently studied only for plane wave light beams [2–5] although in [6] some generalized Stokes parameters were introduced for examining light beams with arbitrary wavefronts within classical statistical optics.

But recently in a series of papers [15–17] a new formalism was proposed for a description of polarization structure of multimode quantum light fields using the polarization SU(2) symmetry and a related concept of the P-quasispin which generalizes the Stokes vector notion at the quantum level and is intimately related to the Stokes operators defined in [18]. This approach allows us to gain a new insight into the polarization structure of light and quantum mechanisms of its depolarization.

  

The aim of this paper is to give a consistent description of the polarization structure of quantum light beams with arbitrary wavefronts using the above mentioned formalism of $P$-(quasi) spin. Specifically, we will show that quantum light beams may be thought as a mixture of familiar polarized photons and specific unpolarized biphoton clusters. New states of unpolarized light, revealed within this approach [15–17], are examined. In addition we discuss briefly some applications of the results obtained.

## 2. Preliminaries: polarization SU(2) invariance and P-spin of electromagnetic fields

Let us consider the free transverse electromagnetic field with 'm' spatiotemporal modes (as it is adopted in quantum optics) which is described by the vector potential

$$A(\boldsymbol{r},t)=c\sum_{j=1}^{m}(2\pi h/\omega_j V)^{1/2}\{A^{(+)}(j)\exp[\mathrm{i}(\boldsymbol{k}_j\boldsymbol{r}-\omega_j t)]+A^{(-)}(j)\exp[-\mathrm{i}(\boldsymbol{k}_j\boldsymbol{r}-\omega_j t)]\}$$

$$A^{(-)}(j)=\sum_{\lambda=\pm,3}e^{(\lambda)}(j)a_\lambda^+(j)\qquad A^{(+)}(j)=(A^{(-)}(j))^+\qquad [a_\lambda(j),a_{\lambda'}^+(j')]=\delta_{\lambda\lambda'}\delta_{jj'}\quad(2.1)$$

and the Hilbert space of quantum states $L_{\text{phys}}=\text{Span}\{|\{n_j^\lambda\}>\}$ spanned by the Fock basis vectors

$$|\{n_j^\delta\}>=N(\{n_j^\delta\})\prod_{j=1}^{m}\prod_{\lambda=\pm}[n_j^\lambda!]^{-1/2}(a_\lambda^+(j))^{n_j^\lambda}|0\rangle\qquad a_\lambda(j)|0\rangle=0\qquad(2.2)$$

which are generated by creation operators $a_\lambda^+(i)$ of 'physical' photons with transverse polarizations (helicities $\lambda=+,-$) only (the gauge condition for the transverse radiation field fixing an absence of non-physical temporal and longitudinal photons); from hereon $e^{(\lambda)}(j)$ are the polarization unit vectors adapted to the helicity basis, $e^{(3)}(j)=c\boldsymbol{k}_j/\omega_j$, $V$ is a quantization volume. (We do not use in this paper a more general relativistically invariant formulation admitting also consideration of nonphysical photons since such generalizations do not touch the main points of the following analysis [16, 17].)

The starting point of our analysis is the obvious invariance of standard expressions

$$H=h\sum_{j=1}^{m}\omega_j\sum_{\delta=\pm,3}N_\delta(j)\qquad(2.3a)$$

$$\boldsymbol{P}=h\sum_{j=1}^{m}\boldsymbol{k}_j\sum_{\delta=\pm,3}N_\delta(j)\qquad N_\delta(j)=a_\delta^+(j)a_\delta(j)\qquad(2.3b)$$

for the Hamiltonian $H$ and the momentum $\boldsymbol{P}$ of the field under the transformations [15–17]

$$a_\alpha^+(j)\to\hat{a}_\alpha^+(j)=\sum_{\beta=\pm}u_{\alpha\beta}a_\beta^+(j)\qquad\alpha=+,-\qquad a_\alpha(j)\to\hat{a}_\alpha(j)=(a_\alpha^+(i))^+\qquad(2.4)$$

from the group $\mathrm{U}(2)=\{u=\|u_{\alpha\beta}\|\}$. We note that equations (2.3) admit, in fact, the more vast group $\mathrm{U}(3)\supset\mathrm{U}(2)$ of polarization transformations, but in quantum optics it reduces to the above U(2) group due to the requirement of conservation of the structure (2.2) of the space $L_{\rm phys}=L_F$ on which quantum expectations of physical quantities are calculated [2].

The transformations (2.4) correspond to the U(2) 'rotations' of the polarization unit vector $e^{(\lambda)}(i)$ [14] in a 'polarization spinor space' [15, 18]

$$e^{(\alpha)}(j)\rightarrow \hat{e}^{(\alpha)}(j)=\sum_{\beta=\pm} u_{\beta\alpha}\, e^{(\beta)}(j) \tag{2.5}$$

and, therefore, may be interpreted as specific polarization gauge transformations. The generators of the obtained polarization invariance group U(2) are of the form

$$N=\sum_{j=1}^{m}\sum_{\alpha=\pm} N_\alpha(j) \qquad P_\pm=\sum_{j=1}^{m} P_\pm(j)=\sum_{j=1}^{m} a_\pm^\dagger(j)a_\mp(j)$$
$$P_0=\sum_{j=1}^{m} P_0(j)=(1/2)\sum_{j=1}^{m}[N_+(j)-N_-(j)] \tag{2.6}$$

where $N$ is the total number operator of physical photons and operators $P_\alpha$ are generators of the SU(2) subgroup defining the polarization (P) spin [15–17]. The operators $P_\alpha$ and $N$ satisfy commutation relations

$$[N,P_\alpha]=0 \qquad [P_0,P_\pm]=\pm P_\pm \qquad [P_+,P_-]=2P_0 \tag{2.7}$$

and in the case $m=1$ coincide up to the factor $\frac{1}{2}$ with the Stokes operators $\Sigma_\alpha$ introduced in [18] in the case of one spatiotemporal mode.

We note that operators $P_\alpha$ do not commute with components $S_\alpha$ of the gauge non-invariant (and hence locally non-observable [18]) familiar spin $S=(S_1,S_2,S_3)$ of the electromagnetic field which defines the field transformations with respect to the SO(3) group of rotations in the usual 3-space. Indeed, the components $S_\alpha$ are expressed in terms of the $A(r,t)$ Fourier components as follows

$$S_\alpha=-\mathrm{i}\sum_{j=1}^{m}\sum \varepsilon_{\alpha\beta\gamma}A_\beta^{(-)}(j)A_\gamma^{(+)}(j) \tag{2.8}$$

where $\varepsilon_{\alpha\beta\gamma}$ is the fully antisymmetric tensor, $\varepsilon_{123}=1$, $A_a^{(\pm)}(j)$ is the projection of $A^{(\pm)}(j)$ on the $a$th axis of a fixed spatial frame [17–18]. Then, from (2.1), (2.6), (2.8) one gets

$$[P_0,S_a]=1/2\sum_{j=1}^{m}\{e_a^{(+)}(j)[a_3^\dagger(j)a_+(j)+a_-^\dagger(j)a_3(j)]$$
$$+e_a^{(-)}(j)[a_3^\dagger(j)a_-(j)+a_+^\dagger(j)a_3(j)]\} \tag{2.9a}$$

$$[P_\pm,S_a]=\mp\sum_{j=1}^{m}\{2e_a^{(3)}(j)a_\pm^\dagger(j)a_\mp(j)+e_a^{(\pm)}(j)[a_\pm^\dagger(j)a_3(j)-a_3^\dagger(j)a_\pm(j)]\} \tag{2.9b}$$

where $e_a^{(\lambda)}(j)$ is the projection (directing cosine) of $e^{(\lambda)}(j)$ on the $a$th axis. In the case of plane wave beams, when all $e_a^{(3)}(j)=\delta_{a3}$, $a=1,2,3$, $e_3^{(\pm)}(j)=0$, from (2.9) one finds equations

$$\exp(\mathrm{i}\varphi S_3)P_\alpha\exp(-\mathrm{i}\varphi S_3)=\exp(\mathrm{i}\alpha\varphi)\,P_\alpha \qquad \alpha=0,+,- \tag{2.10}$$

defining transformations of $P$-spin components under rotations around the light beam axis.

## 3. Polarization quantum optics. Unpolarized quantum light

Equations (2.6) imply a physical meaning of different components $P_\alpha$ as certain quantities measurable in photon counting experiments (with broad–band detectors). In particular, the total helicity $2P_0$ of the field is the difference $(N_- - N_-)$ of the right and left-handed photon numbers and Hermitian operators $2P_1=(P_+ + P_-)$ and $2P_2=i(P_+ - P_-)$ determine (cf [3]) differences $(N_x - N_y)$ and $(N_x - N_y)$ of photon numbers with two pairs of orthogonal linear polarizations which are connected with the helicity basis by the unitary transformations

$$a_x^+(j)=\{a_-^+(j)-a_+^+(j)\}/\sqrt{2} \qquad a_y^+(j)=i\{a_-^+(j)+a_+^+(j)\}/\sqrt{2} \tag{3.1a}$$

$$\tilde{a}_x^+(j)=\{a_x^+(j)+a_y^+(j)\}/\sqrt{2} \qquad \tilde{a}_y^+(j)=\{-a_x^+(j)+a_y^+(j)\}/\sqrt{2} \tag{3.1b}$$

which are implemented with the help of phase plates and (for (3.1*b*)) the rotation by angle $\varphi=\pi/4$ [3, 12].

Besides, in the case of the monochromatic plane waves quantum expectations $\langle P_\alpha\rangle$ are proportional to the Stokes parameters $s_\alpha$: $s_1=2\langle P_2\rangle$, $s_2=-2\langle P_0\rangle$, $s_3=-2\langle P_1\rangle$ which are expectation values of the appropriate Stokes operators $\Sigma_\alpha$ [18]. Therefore, one can consider that in general cases $\langle P_\alpha\rangle$, $\langle N\rangle$ determine the polarization degree $\deg P$ of quantum light beams with arbitrary frequencies and wave vectors by the relation

$$\deg P=2[(\langle P_1\rangle)^2+(\langle P_2\rangle)^2+(\langle P_0\rangle)^2]/\langle N\rangle \tag{3.2}$$

which is similar to (1.1).

The quantum averages $\langle|P^2|\rangle=\bar{p}(\bar{p}+1)$ of the $SU(2)_{\text{pol}}$ Casimir operator $P^2=1/2(P_+P_- + P_-P_+)+(P_0)^2$ are connected by the relation

$$\bar{p}(\bar{p}+1)-[\langle N\rangle \deg P/2]^2=\sigma_{p_0}+\sigma_{p_1}+\sigma_{p_2} \tag{3.3}$$

with the variances $\sigma_{p_\alpha}=\langle|P_\alpha^2|\rangle-(\langle|P_\alpha|\rangle)^2$ determining polarization noises [2, 17] and a radial uncertainty measure for the SU(2) operators [19]. Further, calculating the eigenvalue $p(p+1)$ of the operator $P^2$ on the subspace of one-photon states we find $p(p+1)=\frac{3}{4}$, i.e. the physical photon should be ascribed the value $p=\frac{1}{2}$, as against $S=1$ for the ordinary spin as follows from (2.8), (2.9):

$$\langle\varphi|S^2|\psi\rangle=\langle\varphi|(S_1^2+S_2^2+S_3^2)|\psi\rangle$$

$$=\langle\varphi|\left[4\sum_{j=1}^{m}\sum_{l=1}^{m}(\boldsymbol{k}_j\cdot\boldsymbol{k}_l)/\omega_j\omega_l P_0(j)P_0(l)+N\right]|\psi\rangle, \; |\varphi\rangle, |\psi\rangle\in L_{\mathrm{F}}. \tag{3.4}$$

(We note that (3.4) is valid only for states describing physical photons.)

This fact allows us to identify $P$-spin of one-photon states with the so-called effective spin [4, 11, 20] clarifying simultaneously a physical meaning of the latter one as a specific 'radial' measure (cf (3.3)) of polarization properties of light beams with arbitrary wavefronts. At the same time the ordinary spin $S$ has no such direct connections with proper polarization properties of light related to counting photons with definite polarizations (because of (2.9)) though it is an adequate tool for describing 'rotation' properties of appropriate experiments [12]. Specifically, from (2.1) and (2.8) one easily finds relations

$$[S_\alpha, a_\pm^+(j)]=\pm a_\pm^+(j)e_\alpha^{(3)}(j)\mp a_3^+(j)e_\alpha^{(\pm)}(j) \tag{3.5a}$$

$$[S_\alpha, a_3^+(j)]=-a_-^+(j)e_\alpha^{(+)}(j)+a_+^+(j)e_\alpha^{(-)}(j) \tag{3.5b}$$

$$[S_\alpha, A_\beta^{(-)}(j)] = -i \sum \varepsilon_{\alpha\beta\gamma} A_\gamma^{(-)}(j) \tag{3.5c}$$

which together with (2.9) and (2.10) specify angle dependences of the measurement results on mutual spatial arrangements of light beams and different measurement devices (polarization analysers, detectors, etc.). Morerover, the SO(3) group formalism related to the ordinary spin allows us to expand familiar correlation tensors

$$G^{(S,p)}_{i_1 \ldots i_s; j_1 \ldots j_p}(\{r_\alpha, t_\alpha; r_g, t_g\}) = \langle E_{i_1}^{(-)}(r_1 t_1) \ldots E_{i_s}^{(-)}(r_s, t_s) E_{j_1}^{(+)}(r_1, t_1) \ldots E_{j_p}(r'_p, t'_p) \rangle,$$

$$E = -c^{-1} \partial A / \partial t$$

in sums of the SO(3) irreducible tensors (expectation values of multipole or related to them polarization operators) which possess well-defined transformation properties with respect to the spatial SO(3) group. For example, similar expansions were given by Roman in [6] for $G_{ij}^{(1,1)}(\ldots)$ in order to define some generalized Stokes parameters. We also note that expectation values of the P-spin components can be determined via the Fourier transformation of $G_{ij}^{(1,1)}(\ldots)$ as it follows from (2.1) and (2.6).

Therefore one may use P-spin vector $(P_\alpha)$ as an adequate tool for studying proper polarization properties of quantum light fields in parallel to the usual apparatus of the correlation functions which give an adequate spatiotemporal description of all properties of light beams [2]. But unlike the latter, use of the P-spin formalism allows us to gain a deeper insight into the inner nature of the polarization structure of light beams with arbitrary wavefronts.

Indeed, as was shown in [17], one can decompose the Fock space $L_F$ spanned by the vectors (2.2) into the direct sum

$$L_F = \sum_{p,\pi} \oplus L(p\pi) \tag{3.6}$$

of infinite-dimensional subspaces $L(p\pi)$ which are specified by eigenvalues $p, \pi$ of the $P$-spin and $P_0$ respectively and spanned by basis vectors $|p\pi; n, \lambda\rangle$ of the form

$$|p\pi; n, \lambda\rangle = \sum C(\{\alpha_i, \beta_{ij}, \gamma_{ij}\}) \prod_i (a_\pm^\dagger(j))^{\alpha_i^\pm} \prod_{i,j} (Y_{ij}^\dagger)^{\beta_{ij}} (X_{ij}^\dagger)^{\gamma_{ij}} |0\rangle \tag{3.7a}$$

where the summation in (3.7*a*) is constrained by the conditions

$$\sum \alpha_i = 2|\pi| \qquad \sum \beta_{ij} = 2(p - |\pi|) \qquad \sum \gamma_{ij} = n - 2p \tag{3.7b}$$

and either all $\alpha_i^+ = 0$ (for $\pi < 0$) or all $\alpha_i^- = 0$ (for $\pi > 0$). The coefficients $C(\ldots)$ in (3.7) are determined from the defining equations

$$P^2 |p\pi; n, \lambda\rangle = p(p+1) |p\pi; n, \lambda\rangle \qquad P_0 |p\pi; n, \lambda\rangle = \pi |p\pi; n, \lambda\rangle$$

$$N |p\pi; n, \lambda\rangle = n |p\pi; n, \lambda\rangle \tag{3.8}$$

and some other equations for fixing an extra complex label $\lambda$ [17].

Operators

$$Y_{ij}^{+}=1/2[a_{+}^{+}(i)a_{-}^{+}(j)+a_{-}^{+}(i)a_{+}^{+}(j)] \qquad (3.9a)$$

and

$$X_{ij}^{+}=[a_{+}^{+}(i)a_{-}^{+}(j)-a_{-}^{+}(i)a_{+}^{+}(j)] \qquad (3.9b)$$

in (3.7*a*) are solutions of the operator equations

$$[P_0, Y_{ij}^{+}]=0 \qquad [P_\alpha, X_{ij}^{+}]=0 \qquad \alpha=0, +, - \text{ (or } 0,1,2) \qquad (3.10)$$

and may be interpreted as creation operators of $P_0$-scalar and $P$-scalar biphoton kinematic clusters respectively, i.e. two-photon pairs with fixed phase correlations of two or four waves. Therefore, in general, the states (3.7) describe light beams representing a mixture of both usual photons and $P$- and $P_0$-scalar biphotons [17]. Indeed, these states are generated by the action of $(Y_{ij}^{+})^{\beta_{ij}}$ and $(X_{ij}^{+})^{\gamma_{ij}}$ on the vectors $|v\rangle_\pi=\text{Span}\ \{||\pi|\pi; 2|\pi|, \lambda\rangle\}$ describing completely polarized states of light and having 'vacuum' properties: $Y_{ij}|v\rangle_\pi=0=X_{ij}|v\rangle_\pi$ with respect to operators $Y_{ij}=(Y_{ij}^{+})^{+}$, $X_{ij}=(X_{ij}^{+})^{+}$. We, however, note that biphotons $Y_{ij}^{+}$ exist for any number $m$ of spatiotemporal modes whereas biphotons $X_{ij}^{+}$ only for $m\geqslant 2$. We also emphasize that in contrast to the usual photon operators $a_\lambda^{+}(j)$, $a_\lambda(j)$ the operators $X_{ij}^{+}$, $X_{ij}$, $Y_{ij}^{+}$, $Y_{ij}$ satisfy not the canonical commutation relations but trilinear commutation relations for quanta of parastatistical fields [17]. However, using a generalized Holstein–Primakoff mapping [20] one can construct from them some operators $W_a^{+}$, $W_a$ obeying canonical commutation relations [16, 17] and representing peculiar two-photon 'optical atoms' (cf [21]). For example, the operators

$$W^{+}=Y_{11}^{+}[N/2+I+|P_0|]^{-1/2} \qquad |P_0|=\sqrt{P_0^2} \qquad W=(W^{+})^{+} \qquad (3.11)$$

obtained in such a manner satisfy the relations: $[W, W^{+}]=I$, $W^{+}W=N/2-|P_0|$ ($I$ stands for the unity operator) describing particle-like two-photon excitations with a specific collective binding energy $2\omega|\pi|$ on the invariant subspaces $L'(\pi\neq 0)=\text{Span}\ \{|\{n^{+}, n^{-}\}\rangle$: $P_0|\{n^{+}, n^{-}\}\rangle=\pi|\{n^{+}, n^{-}\}\rangle\}$. This construction, in a sense, realizes the method of fusion by de Broglie [22].

Further, the decomposition (3.6) is invariant with respect to the Lie algebra so* $(2m)$ generated by operators $X_{ij}^{+}$, $X_{ij}$ [17]. Therefore states $|\varphi\rangle$ belonging to a subspace $L(p\pi)$ with given $p$, $\pi$ at an initial time will be in it for the time evolution governed by the free Hamiltonian (2.3*a*) or interaction Hamiltonians $H_{int}=H'_{int}(\{X_{ij}^{+}, X_{ij}\})$. Extending the algebra so* $(2m)$ by adding operators $Y_{ij}^{+}$, $Y_{ij}$ we get the algebra $u(m, m)$ associated with interaction Hamiltonians $H_{int}=H''_{int}(\{Y_{ij}^{+}, Y_{ij}; X_{ij}^{+}, X_{ij}\})$ which remain invariant for time evolution subspaces [17]

$$L'(\pi=\sum_{p\geqslant|\pi|} L(p\pi).$$

The simplest examples of such interaction Hamiltonians are given by expressions

$$H'_{int}=\sum_{i<j}(g_{ij}X_{ij}^{+}+g_{ij}^{*}X_{ij}) \qquad (3.12a)$$

$$H''_{int}=H'_{int}+H'''_{int} \qquad H'''_{int}=\sum_{i\leqslant j}(f_{ij}Y_{ij}^{+}+f_{ij}^{*}Y_{ij}) \qquad (3.12b)$$

and describe some specific parametric processes [16, 17].

The decomposition (3.6) also implies a new classification of the polarization states of quantum light fields from the physical viewpoint [15–17]. Specifically, for the states $|\pi=0\rangle \in L'(0)$ and $|p=0\pi=0\rangle \in L(00)$ from (2.6), (2.8), (3.7)–(3.10) we find $\langle|P_\alpha|\rangle=0$, $\langle|S_\alpha|\rangle=0$ for all $\alpha$ that is a characteristic property of unpolarized light (cf [2, 3]). Besides the calculations [16] showed that for these states, correlation tensors $G_{ij}^{(1,1)}(\boldsymbol{r}, t; \boldsymbol{r}, t)$ are expanded in a sum of the SO(3) scalar and quadrupole tensors and have a form similar to that which describes completely unpolarized light beams (with, in general, arbitrary wavefronts) in classical statistical optics [6].

But unlike the classical (chaotic) unpolarized light, for the states $|p=0\pi=0\rangle$ and $|\pi=0\rangle$ we have additional characteristics of light depolarization which follow from equations (3.7)–(3.10) and are expressed in terms of higher moments for $P_\alpha$:

$$\langle|(P_0)^n|\rangle=0 \qquad \langle|(P_\alpha)^n|\rangle\neq 0 \qquad \alpha=1,2, n\rangle 1 \qquad \text{for } |\rangle \in L'(0) \tag{3.13a}$$

$$\langle|(P_\alpha)^n|\rangle=0 \qquad \alpha=0,1,2, n>1 \qquad \text{for } |\rangle \in L(00) \tag{3.13b}$$

showing the complete absence of appropriate polarization noises of any order measured by noises of difference currents in schemes of photodetectors with polarization analysers (cf [9]); herewith, as it follows from (2.10), for axial light beams (with parallel wave vectors $\boldsymbol{k}_i$) results of measurements do not depend on rotations of analysers around beam axis.

Thus, for the states $|\rangle \in L(00)$ all proper polarization properties are identical with those for vacuum state $|0\rangle$, but unlike the latter the light intensity in these states is not equal to zero. Consequently, they may be recognized as states describing absolutely unpolarized light while the states $|\rangle \in L'(0)$ have a hidden polarization structure revealed in measurements of linear polarization noises. Moreover, the states $|p=0\pi=0\rangle$ minimize both the aforementioned 'radial' and other uncertainty relations for the su(2) operators as well as form the infinite-dimensional space of quantum states on which three non-commuting operators $P_\alpha$ behave then as $c$-numbers. However, we have no analogous relations (of the (3.13) type) for components $S_\alpha$ of the ordinary spin as follows from (3.4), (3.5), (3.9).

Therefore, states $|\varphi\rangle \in L'(0)$ generated by biphotons $Y_{ij}^+$, $X_{ij}^+$ and $|\psi\rangle \in L(00) \subset L'(0)$ generated only by biphotons $X_{ij}^+$ describe new types of unpolarized light due to strong quantum phase correlations rather than random mixing light beams as is the case for classical unpolarized light [2, 3]. Examples of such states are yielded by generalized coherent states related to interaction Hamiltonians (2.12). Specifically, generalized coherent states of the SO*(2$m$) group orbit type

$$|\mu_{ij}\rangle_x = \exp\left[\sum (\mu_{ij} X_{ij}^+ - \mu_{ij}^* X_{ij})\right]|0\rangle \tag{3.14}$$

are generated by $H'_{\rm int}$ (see, e.g., [16, 17] where they are discussed together with some related models) whereas $H'''_{\rm int}$ produces the $Sp(2m, R)$ generalized coherent states

$$|\gamma_{ij}\rangle_Y = \exp\left[\sum (\gamma_{ij} Y_{ij}^+ - \gamma_{ij}^* Y_{ij})\right]|0\rangle \tag{3.15}$$

coinciding in the cage $m=l$ with two-mode squeezed vacuum states [23] and describing, in particular, so-called twin-photon beams [9].

All other subspace $L(p\pi)$, $L'(\pi)$, $|\pi|>0$, describe states of partially polarized quantum light (see [17] where we also examined various types of polarization generalized coherent state of light including those which are eigenfunctions of the operators $P_0$, $P^2$, $X_{ij}$, $Y_{ij}$ and generalize the Agarwal's pair coherent states [24]). We also note that acting by the group displacement operators from (3.14), (3.15) on usual multimode Glauber coherent states $|\alpha_i^+, \alpha_i^-\rangle$, $\alpha_i^\pm \neq 0$, we get, in the general case, states of partially polarized light which contains (for special values of parameters $\alpha_i^\pm$) a subclass of states corresponding to unpolarized light. Specifically, all states related in such a manner to $|\alpha^+, \alpha^-\rangle$ display properties of usual unpolarized light when $|\alpha^+| = |\alpha^-|$ (cf [7]). But relations of the (3.13) type are not available for them.

Thus, our analysis displays inner mechanisms of the light depolarization at the quantum level (cf [25] where a conjecture was uttered about the quantum nature of unpolarized light)) by contrast with the generally accepted viewpoint [3] that randomization is the only way of obtaining unpolarized light. Besides, as follows from (3.7), (3.8), the $P$-spin formalism yields some natural measurable characteristics of light depolarization, namely, degrees $\operatorname{dep} P = (1 - 2\bar{p}/\bar{N})$ and $\operatorname{dep} P_0 = (1 - |2\bar{\pi}|/\bar{N})$ of the content of $P$-scalar and of $P_0$-scalar bihpotons where $\bar{p}$, $\bar{\pi}$, $\bar{N}$ stand for expectation values of appropriate operators; herewith $\bar{p}$ is determined from (3.3). Evidently, $\operatorname{dep} P_0$ is connected with the well known degree of circular polarization $|\langle N_+\rangle - \langle N_-\rangle|/\langle N\rangle$ whereas $\operatorname{dep} P$ provides a new quantitative characteristic of polarization structure of light related to measurements of noises.

We also note that analysis above can be extended by considering a modification of the decomposition (3.6) where instead of $P_0$ any operator $P_\alpha$, $\alpha = 1, 2$, corresponding to a linear polarization basis is diagonalized. Such an extension leads to new states of quantum unpolarized light generated by $P_1$—or $P_2$—scalar biphotons of the (3.9*a*) type and having characteristics similar to those described by (3.13*a*) but with some peculiarities concerning their 'rotation' properties determined by (2.9), (2.10) and (3.2). Specifically, the analogues of (3.13*a*) are valid only for the situation when analyser axes coincide with those determining appropriate linear polarizations. A more detailed analysis of this question and related topics will be given elsewhere.

## 4. Applications and conclusion

The $P$-spin formalism and non-classical states of light described above have several potential applications; one, for example, is in optical communication theory [26–28]. Specifically, the properties (3.10) of states $|\psi\rangle \in L(00)$ and $|\psi\rangle \in L'(0)$ and the decomposition (3.6) appear to be promising for designing the quantum channels of communication systems [26]. Such communiction channels are realized by light beams using both amplitude and phase modulations for encoding information [26, 27]. But polarization methods for its encoding appear to be more preferable because of certain (mainly energetics) reasons. We sketch a scheme using quantum unpolarized light within such an approach, following [27].

For discrete channels their efficiency is usually estimated with the aid of the conditional error probability $P_e[m \neq \hat{m}]$ where $m$ is an input message and $\hat{m}$ is the appropriate output one [26]. Then, using, states $|\psi_0\rangle \in L'(0)$ for transmitting the logical 0 and states $|\psi_1\rangle \in L'(\pi \neq 0)$ for transmitting the logical 1 in binary discrete channels, one can use the results obtained above for optimizing $P_e[\ldots]$ (cf [26]). For this end it is also of interest to estimate the information capacity [28] of the states $|\psi\rangle \in L'(0)$ as compared with that of other quantum states. This scheme may find an implementation, for example, in biocomputing design [17, 27].

For other lines of possible applications of the results we point out precise measursents of polarization (chiral) properties of anisotropic media and studies of the interaction of different kinds of quantum unpolarized light with optically active biological macromolecules (cf [29]). We also note that the above formal constructions (especially, the decomposition (3.6) and the mapping (3.11)) may be used in other physical theories with internal SU(2) symmetries.

In conclusion we emphasize that the above results give a deeper insight into the polarization structure of light allowing determination of new unusual polarization states in quantum optics, in particular, twin-photon states with hidden polarization structure and absolutely unpolarized states of *P*-scalar light. In a sense, the above results yield necessary prerequisites for developing a quantum description of unpolarized light waves (cf [30]) whose existence has not yet an adequate solution within classical optics [3, 30]. Besides, we established some interrelations between proper polarization (*P*-spin) and rotation (spin) characteristics of light fields (equations (2.9), (2.10)) that allows us to examine the behaviour of polarization characteristics and dependence on rotation of measurement devices with respect to light beam directions (axes).

All this opens some possibilities in setting new optical experiments related, in particular, to 'hidden' variables and EPR paradox [1, 4, 10], polarization chaos, spontaneous symmetry breaking and bistability [7, 13, 25], 'optical atoms' and polarization solitons [8, 20], reduction of quantum noises [9, 11, 23], etc. We are planning to discuss these topics as well as some practical schemes and mechanisms of production of quantum unpolarized light (specifically, due to the obvious fact that states (3.14), (3.15) are decomposed into products of those describing beams of two-mode squeezed light obtained in simple schemes of parametric scattering (cf [9, 11, 23])) and its standard squeezing properties (in terms of the field quadrature components) and interactions with material media in forthcoming papers (see also [15–17, 31, 32]).

The author thanks V P Bykov, D N Klyshko and A V Masalov for stimulating discussions and G S Agarwal, V L Derbov and S I Vinitsky for interest in this work. The work was supported in part by the Ministry of Science, High School and Technical Policy of Russia, Grant Numbers 07-14 and 107-51-74.

## References

[1] Clauser J F and Shimony A 1978 *Rep. Prog. Phys.* **41** 1881
[2] Perina J 1984 *Quantum Statistics of Linear and Nonlinear Optical Phenomena* (Dordrecht: D Reidel)
[3] Gerrard A and Burch J M 1975 *Introduction to Matrix Methods in Optics* (London: Wiley)
[4] Bohm A 1986 *Quantum Mechanics: Foundations and Applications* (New York: Springer)
[5] Dubovik V, Markovski B and Vinitsky S (eds) 1989 *Topological Phases in Quantum Theory* (Singapore: World Scientific)
Vinitsky S I, Derbov V L, Dubovik V M, Markovski B W and Stepanovskii Yu P 1990 *Uspekhi Fiz. Nauk* **160** 1 [Russian]
[6] Roman P 1959 *Nuovo Cimento* **13** 974
Barakat R 1963 *J. Opt. Soc. Amer.* **53** 317
[7] Ritze H-H and Bandilla A 1980 *Phys. Lett.* **78A** 447
[8] Tratnik M V and Sipe J E 1987 *Phys. Rev. Lett.* **58** 1104
[9] Heidmann A, Horowicz R J, Reynaud S, Giacobino E, Fabre C and Camy C 1987 *Phys. Rev. Lett.* **59** 2555
[10] Shih Y H and Alley C O 1988 *Phys. Rev. Lett* **61** 2921
[11] Klyshko D N 1989 *Phys. Lett.* **140A** 19

[12] Simon R and Mukunda N 1990 *Phys. Lett.* **138A** 474
[13] Buller G S, Campbell R J and Walker A C 1990 *Opt. Commun.* **75** 93
[14] Strazhev V I, Fyodorov F I and Shkolnikov P L 1982 *Dokl. AN BSSR* **26** 593
[15] Karassiov V P and Puzyrevsky V I 1989 *J. Sov. Laser Res.* **10** 229
[16] Karassiov V P 1991 *J. Sov. Laser Res.* **12** 147
[17] Karassiov V P 1991 *J. Sov. Laser Res.* **12** 431
[18] Jauch J M and Rohrlich F 1959 *Theory of Photons and Electrons* (London: Addison-Wesley)
[19] Delbourgo R 1977 *J. Phys. A: Math. Gen.* **10** 1837
[20] Holstein T and Primakoff H 1940 *Phys. Rev.* **58** 1098
Curtright T L and Zachos C K 1990 *Phys. Lett.* **243B** 237
[21] Woerdman J P and Spreeuw R J C 1990 *Analogies in Optic and Microelectronics* (Dordrecht: Kluwer) p 135
[22] De Broglie L 1954 *Thèorie generale des particules á spin: Methode á fusion* (Paris: Gauthier-Villars)
[23] Loudon R and Knight P L 1987 *J. Mod. Optics* **34** 709
[24] Agarwal G S 1988 *J. Opt. Soc. Amer.* B5 1940
[25] Lipkin H J 1973 *Quantum Mechanics. New Approaches to Selected Topics* (Amsterdam: North-Holland)
[26] Helstrom C W, Liu J W S and Gordon J P 1970 *Proc. IEEE* **58** 1578
[27] Karassiov V P and Puzyrevsky V I 1992 *Symmetry methods in Physics Proc. V Workshop*, Obninsk, July 1991 (Obninsk. Inst. Phys. Power Eng. Publ.) p 108
[28] Yamamoto Y and Haus H A 1986 *Rev. Mod. Phys.* **58** 1001
[29] Mason S F 1982 *Molecular Optical Activity and the Chiral Discrimination* (Cambridge: University Press)
[30] Hepp H and Jensen H 1971 *Sitzber. Heidelberg. Acad. Wiss. Mat.-Naturw.* Kl. **4** Abh. 89
[31] Karassiov V P, Prants S V and Puzyrevsky V I 1990 *Interaction of Electromagnetic Field with Condensed Matter* ed N N Bogoliubov, Jr., A S Shumovsky and V I Yukalov (Singapore: World Scientific) p 3
Karassiov V P and Puzyrevsky V I 1991 *Trudy FIAN* **211** 161 (Russian)
[32] Bykov V P 1991 *Uspekhi Fiz. Nauk* **161** 145 (Russian)